Attitude Determination for the Shuttle Radar Topography Mission

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ABSTRACT

The Shuttle Radar Topography Mission (SRTM) is the first mission to provide high accuracy near-global topographic coverage of the Earth's land surface using a long-baseline interferometry approach. It uses a synthetic aperture radar instrument to produce a digital elevation map with 16 m absolute vertical height accuracy at 30 meter postings. The mission involves a large space structure (60 meter mast) deployable from the Shuttle payload bay and requires precision attitude and position determination to arc-minute and mm levels. Onboard flight sensors include electro-optical metrology sensors, a target tracker, a star tracker, gyros, and GPS receivers. An onboard estimator has been designed to determine the attitude of the outboard radar antenna relative to the inboard (main) antenna for mast deployment verification and alignment. Ground processed system identification of the mast frequency and damping is implemented for proper tuning of the Shuttle attitude control system. Estimators for post-flight reconstruction of the attitudes and positions of the outboard and inboard antennas have been designed and simulated. A description of the system architecture, error budgets, and the major issues associated with attitude determination and sensor measurements involving large space structures are also provided.

1. Introduction

The Shuttle Radar Topography Mission (SRTM) is a project jointly sponsored by NASA and the National Imagery and Mapping Agency (NIMA) with the goal of generating high accuracy global topographic maps. SRTM is scheduled for launch in September 1999 for an 11-day Space Shuttle mission. It will provide coverage of 80% of the earth's land surface based on data acquired during flight. It will collect enough radar measurements and onboard sensor data for a digital model of the Earth that will satisfy the requirements of 16 meter absolute vertical height accuracy, and 30 meter postings. The SRTM coordinates and in-flight antenna orientation are shown in Fig 1. These results will provide significant improvement over existing global topographic maps.

Spaceborne synthetic aperture radar (SAR) has demonstrated several advantages over conventional imagery, such as independence from solar illumination, cloud cover, and forest canopies. The Spaceborne Imaging Radar/X-band Synthetic Aperture Radar instruments which flew on the Space Shuttle in 1994, gathered imagery over pre-designated target sites[1]. It had demonstrated repeat-pass interferometry, where imagery of a target was obtained on repeat orbits (the difference in positions on each pass forms the necessary baseline). Unlike other spaceborne SAR platforms, SRTM will operate as a fixed-baseline mapping interferometer, which will effectively remove large error sources inherent with the repeat-pass technique, such as temporal decorrelation and baseline uncertainties[2].

The SRTM architecture consists of a 60 meter mast and supporting mechanisms to deploy an "outrigger" (outboard) radar antenna. The main (inboard) antenna remains inside the Shuttle bay. The attitude determination system provides algorithms for the necessary baseline metrology, and determines attitudes and positions of the radar antennas to accuracy of arc-minutes and milli-meter to support mission operations and topography data processing. Platform position and velocity determination is provided by an onboard GPS system. Detailed description of the GPS data processing is not included in this paper.

2. Architecture for Attitude Determination

The attitude determination system architecture is depicted in Fig 2. The baseline attitude has two primary components: the inboard antenna attitude and

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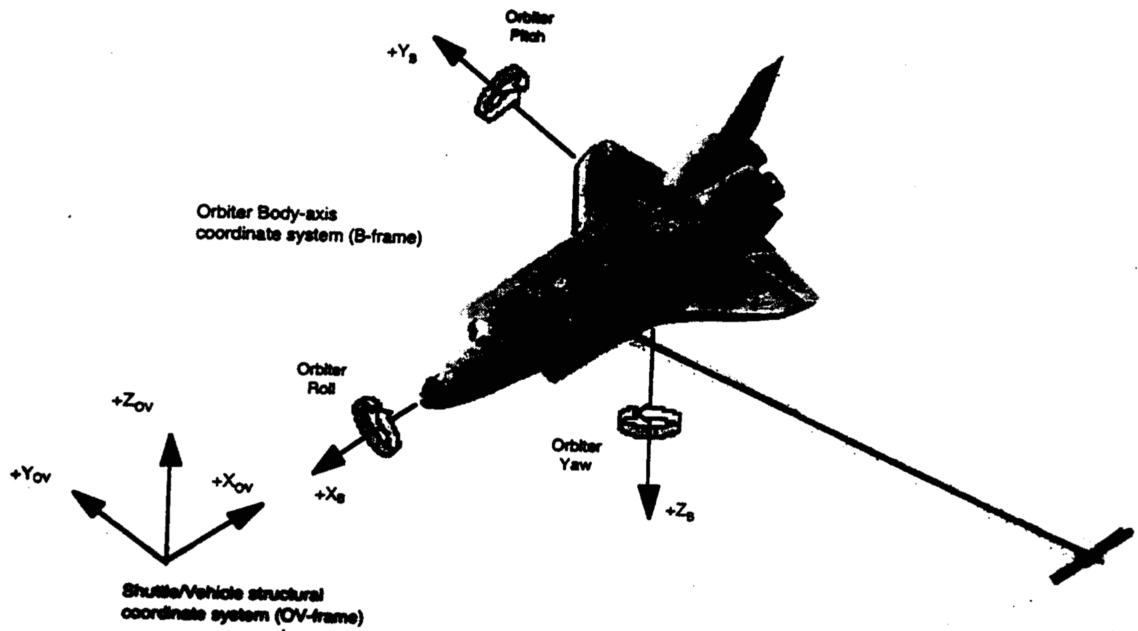


Fig 1 SRTM coordinates and flight configuration

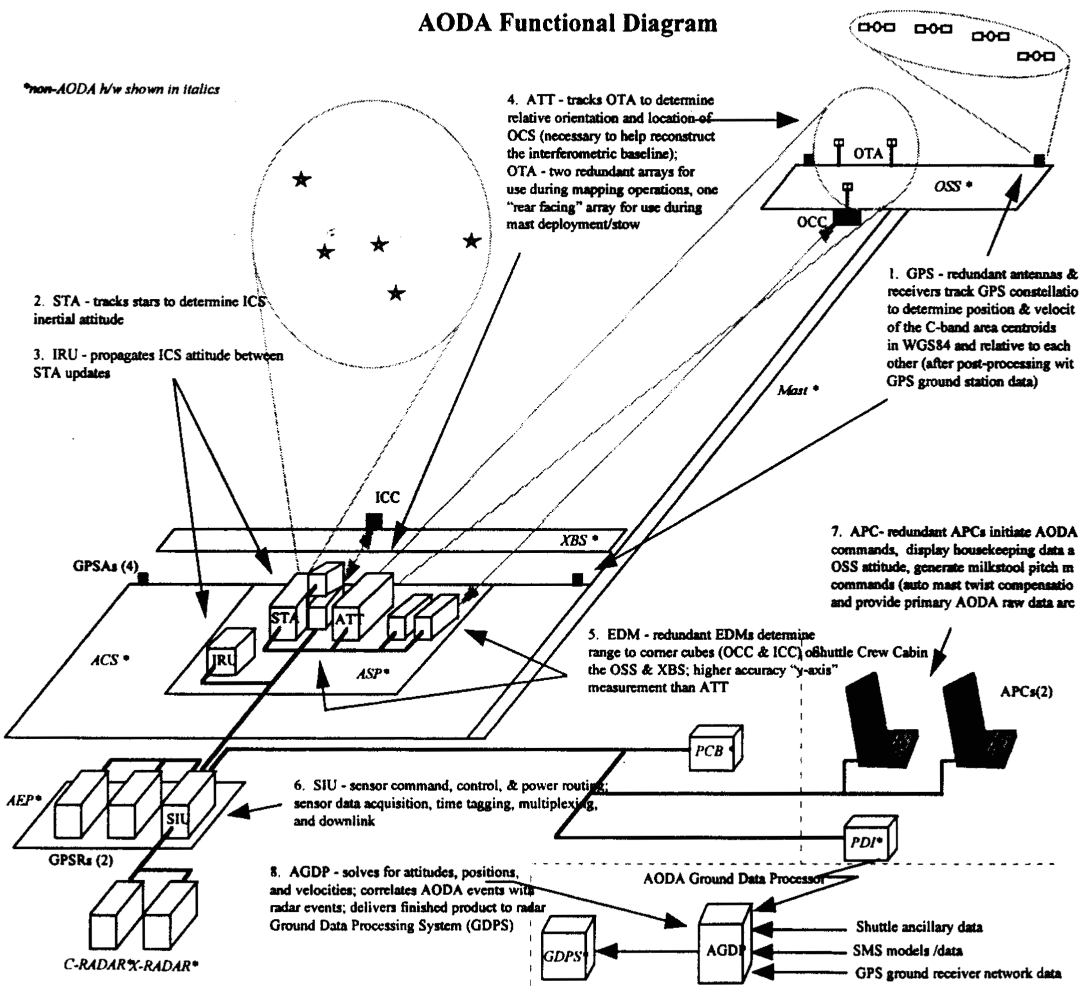


Fig 2 SRTM Attitude Determination Configuration

the relative motion in six degree-of-freedom between the two radar antennas. The inboard attitude is measured by a Star Tracker Assembly and an Inertial Reference Unit (IRU). The star tracker is the Lockheed Martin Autonomous Star Tracker (AST-201). It has a 8.8° x 8.8° field-of-view and employs a star catalog covering 16,000 stars. It performs autonomous acquisition with no a-priori attitude knowledge and typically tracks 30-50 stars. It is capable of full accuracy for rates of 0.5°/s and degraded accuracy for rates as high as 2°/s. The tracker will output quaternions representing its boresight's inertial attitude at 1 hz. The IRU is a Teledyne Dry Rotor Inertial Reference Unit (DRIRU-II) which outputs angular increments also at 1 hz [3]. This particular unit has flown twice on the space Shuttle. The IRU measurements can further refine the attitude estimate during data processing. Note that the star tracker, IRU, and GPS sensors are necessary because the Shuttle guidance and navigation systems cannot provide the required accuracy and also significant thermal distortions are expected between the Shuttle guidance platform and the SRTM inboard antenna.

To determine the outboard antenna's attitude and position relative to the inboard antenna, a target tracker is used to continuously view the outboard from its inboard location. The tracker, developed by JPL in the 1980's to serve as a high precision star tracker for pointing applications, was flown on the Shuttle twice and has demonstrated sub-arcsecond accuracy in-flight. However, significant modifications has been made to accommodate the SRTM architecture, such as adding a new corrective lens to focus at a distance of 60 meters. The tracker will track three red (635 nm) light emission diode (LED) targets located on the outboard antenna. The LEDs will be mounted on graphite-epoxy "stalks" and separated by 1 meter both laterally and in line of sight. The target tracker will output centroid information in charged couple device (CCD) line and pixel coordinates on all three targets at a rate of 4 hz.

Although the target tracker provides very good accuracy in determining 5 of the 6 outboard antenna degree-of-freedom, it has poor accuracy in determining the line-of-sight component. Hence, it is necessary to add additional metrology instruments capable of measuring the range to the outboard antenna to high accuracy. A commercially available surveying rangefinder, known as an Electronic Distance Meter (EDM) (Leica-Wild DI2002) is used. The EDMs have a fairly small field-of-view (approximately 1.7 mrad), and their target is an array of corner-cube reflectors (40 cm diameter) located on the inboard edge of the outboard antenna. This will allow the two outboard-looking EDMs to still acquire enough signal in the event of large mast excursions. Although rotations of the outboard antenna produced some errors in the range solution, the resulting range accuracy is still better than 2 mm.

Two laptop computers with JPL-developed software will be located in the Shuttle's crew cabin. The software will provide automated control loops

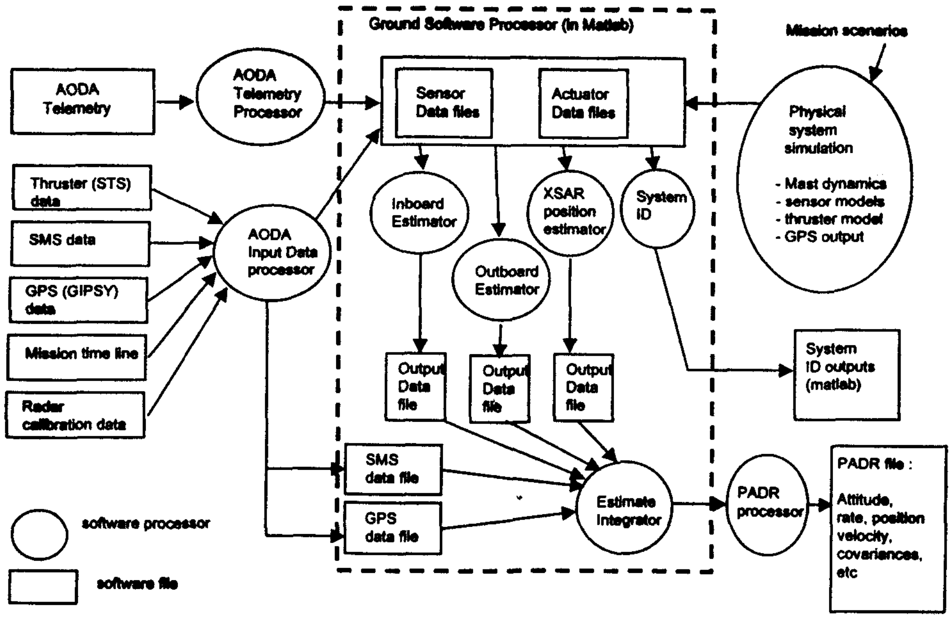


Fig 3 **Ground Software for Attitude Determination**

involving the operation of the target tracker and EDMs. It contains the attitude estimator and the associated display to guide antenna alignment. All the sensor measurement data collected in the 11-day mission will be stored in the computer's hard drives, and transferred once the Shuttle has landed for post-flight attitude reconstruction. Partial sensor data will be downlinked in real-time during the mission for a quick system verification.

The ground attitude reconstruction software has to process telemetry data output by all the sensors above and collected over the 11-day mission. Other data files to be processed simultaneously are the Shuttle thruster firings telemetry, the ground processed GPS data, and the structural deformation data modeled from temperature measurements. The architecture of the ground software is depicted in Fig 3. All the estimators, including modal identification, will read the individual sensor data files, as appropriate. The system identification software will output the identified modal frequencies and damping in a separate data file. The remaining ground software will provide a file containing the attitude and position estimates of the two antennas and their covariances in various coordinate systems at 1 second intervals. A complete matlab simulation of the SRTM/Shuttle orbital motions, the mast dynamics, and sensor measurements has been developed for testing and validation of the ground attitude determination software. Samples of other input data files, such as the sensor telemetry data file, obtained from laboratory hardware-in-the-loop testing, were also used for validating the ground software, particularly for checking data format, interfaces, and pre-processing of data prior to their use by the estimators.

3. Attitude Determination Requirements

In the first 12 hours of the mission, the attitude determination operations will be very closely linked with the deployment of the mast, alignment of the antennas, and optimization of the Shuttle attitude control system. Close interactions between ground operators and the astronaut crew (via the laptop computers) is expected during this period. However, the onboard attitude determination system will operate autonomously once the mapping phase of the mission begins. Automatic control loops running in the onboard software will trigger re-acquisition attempts if target lock fails. Error checking software will alert the astronaut crew and ground mission operation teams in the event of malfunctions.

There are four basic attitude determination requirements for the SRTM mission:

a) Verify successful mast deployment

For safety reasons, successful deployment of the mast must be verified before proceeding with the mapping phase of the mission. If one or more latches do not snap into place completely even when the mast is completely extended, the mast would be susceptible to collapse in response to Shuttle thruster firings. The Shuttle will remain in free drift in a stable gravity-gradient attitude during mast deployment verification. Onboard estimation of deployed mast tip position and attitude errors will enable the astronaut crew and ground teams to verify mast integrity before enabling the Shuttle attitude control system.

b) Guide alignment of the radar antennas

The Space Shuttle crew are expected to be provided with measurements of the mean inboard to outboard mechanical antenna alignment to an accuracy of 50 arcsec about the inboard Y and Z (pitch and yaw) and 100 arcsec about the inboard X (roll), with the presence of mast tip translational velocities of up to 6 cm/s. Even with normal mast deployment, misalignment exists between the inboard and outboard antennas. The static and quasi-static components of pitch and yaw misalignment must be reduced to less than 0.3° before the radar instruments can operate properly. Roll misalignment is not critical since the radar can perform significant electronic beam steering about that axis, and the antenna pattern is an order of magnitude broader along the roll axis than the pitch or yaw axes. The mast tip is equipped with a two-axis "milkstool" actuator that allows for mechanical adjustment of the outboard antenna pitch and yaw alignment. Following mast deployment, the astronaut crew will use attitude estimate displays to guide adjustments with the milkstool mechanism.

c) Conduct system identification of mast characteristics

Due to practical difficulties of performing precision modal tests of a 60 meter deployed mast structure in a 1-g environment, the pre-flight estimates of mast modal frequencies are expected to be accurate to within 20%. Since notch filter settings in the Shuttle's attitude control system are selected to reduce mast response, errors in pre-flight estimates could lead to excessive attitude control response and fuel consumption, thereby shortening the length of the mission. Hence, in-flight modal identification tests will be performed to improve the knowledge of the mast frequencies and damping, and allow for optimization of the Shuttle attitude control system. The tests will involve firing Shuttle thrusters in all axes to excite the mast while sensor measurements are being collected. The knowledge of the mast frequencies following the in-flight modal test is required to be better than 5%.

d) Determine antenna attitude and position post-flight to support topography processing

The errors in baseline angle, baseline length, and platform height have significant impact on total SRTM performance. The baseline roll angle, in particular, is one of the largest error sources (second only to radar phase noise). The requirement is to provide estimation of the interferometric baseline length, attitude, and position at the accuracy of 2 mm, 9 arcsec, and 1 m levels, respectively, throughout the entire mission whenever sensor telemetry data is available.

Expected attitude and position accuracy of the inboard and outboard antennas are shown in error budget tables 1 - 3. Specifically, the attitude of the inboard X,Y,Z axis with respect to the inertial coordinates shall be accurate to [5, 15, 15] arcsec. The position of the outboard origin relative to the inboard shall be accurate to [8.7, 2, 1.5] mm in inboard X,Y,Z coordinates. The attitude determination requirements are specified in relative accuracy as opposed to absolute accuracy. This is because large systematic error biases, such as sensor-to-platform misalignment, or star tracker optical bias, can only be removed by radar calibrations. Such calibration is accomplished by performing radar ground data-takes periodically during the mission over well-surveyed land sites, but the majority will take place over the ocean. Only a few such calibration tie-points are necessary to remove the large systematic biases in the flight system. All accuracies listed in this paper are specified at the 90% (1.6 sigma) level unless specified otherwise.

4. Mast Structure and Dynamics

The mast, developed by AEC-Able, is a composite structure composed of graphite epoxy longerons and battens with steel fittings and titanium cables. When deployed, it forms a rigid structure with primary modal frequencies on the order of 0.1 hz. The mast is designed to provide minimal thermally-induced distortions, allow deployment and retraction (20 min each time), and is capable of being jettisoned in an emergency. However, there are many challenges associated with operating an interferometer with a large deployed mast structure.

The mast alone has very little inherent damping, resulting in the need to add viscous damper struts at the root to prevent resonant instability in response to the Shuttle attitude control system. The mass of the outboard antenna is 128 Kg, and the mast is around 15 Kg. The combined mast and outboard antenna represent a large moment-arm on the Shuttle ordinarily not designed for. Since the SRTM mapping geometry requires that the mast be maintained at a 45° angle with respect to nadir, the resulting gravity-gradient torque is near the worst. A continuous-thrust cold gas propulsion system has been added to the outboard antenna to compensate for this torque and reduce Shuttle fuel consumption.

Mast pointing is affected by several factors. Gravity unloading, launch shifts, and pre-flight assembly and alignment errors result in a quasi-static pointing bias. In-flight thermal distortions of the mast and antenna structures (bending and twisting) create pointing errors with time-constants of tens of minutes. Finally, the mast will dynamically respond to the Shuttle attitude control system thruster firings and astronaut crew activities. Note that mast bending results in relative misalignment between the two antennas (e.g., a 3 cm tip translation results in a 0.1° antenna misalignment). Mast pointing errors impact the attitude determination and radar sensor's capability in both target acquisition and tracking.

5. Onboard Estimator for Outboard Antenna Alignment

The onboard estimator is used to assist the Shuttle crew for the outboard antenna alignment and deployment verification. It uses the target tracker (2.2° x 3.5° FOV) to continually attempt acquisition until 3 targets are found. The output is the centroids of the three targets. Once in track, track windows are maintained about each centroid on the CCD. This would allow faster tracking than full-CCD readouts. Track window position is updated every frame. The tracker runs at 8 hz in order to follow worst-case mast tip velocities of 60 mm/s. However, due to shuttle telemetry bandwidth limitations, every other frame is discarded and the single frame measurement data is nominally read-out at 4 hz. This is more than adequate to Nyquist sample the mast dynamics.

5.1 Estimator Formulation

The estimator contains the appropriate coordinate transformations to relate the outboard antenna attitude and the target tracker measurements. It is designed to be robust to occasional outage of target tracker measurements. A first order linear model of the position and attitude errors with respect to tracker measurements is derived, which leads to a least-squares estimator of the mast tip position and attitude errors. The resulting estimates will be averaged by passing through a 4th order Butterworth filter.

The location and attitude errors of the mast tip is related to the tracker measurements by

$$m = a + A^{-1}\left(l - L^{-1}Dd\right) \qquad M = D^{-1}LA$$

The coordinate transformations are illustrated in Table 4. Taking into account errors in the transformation operators, the baseline error Δm becomes

$$\begin{aligned}\Delta m &= m - m_0 \\ &= \Delta a + A^{-1}\left(I + \alpha^{x}\right)\left[l + \Delta l - L^{-1}\left(I + \lambda^{x}\right)\left(I - \delta^{x}\right)D(d + \Delta d)\right] - A^{-1}\left(l - L^{-1}Dd\right) \\ &= \Delta a + A^{-1}\left[\Delta l - L^{-1}D\Delta d + L^{-1}(Dd)^{x}(\lambda - \delta) - \left(l - L^{-1}Dd\right)^{x}\alpha\right],\end{aligned}$$

where the subscript 0 indicates the nominal value. The parameters $\alpha, \lambda, \text{and} \delta$ expresses small errors in the rotation matrices *A*, *L* and *D* respectively. $\Delta a, \Delta l$ and Δd are errors in the location vectors *a*, *l* and *d*. The errors in the orientation can be summarized in the small angle error μ, $\mu = D^{-1}\left(L\alpha + \lambda - \delta\right)$.

The baseline and small angle errors (Δm, μ) sensitivity equation above can be split in two parts. One that contains the structural error, and one that contains the estimator errors. We call them S_1 and S_2 respectively.

$$\begin{bmatrix}\Delta m \\ \mu\end{bmatrix} = S_1\begin{bmatrix}\Delta a \\ \alpha \\ \Delta d \\ \delta\end{bmatrix} + S_2\begin{bmatrix}\Delta l \\ \lambda\end{bmatrix}$$

The S_1 part is unobservable by the tracker and will cause a bias in the estimate of (Δm,μ). We obtain a least squares estimate for Δl and λ based on the tracker. We obtain the linear model by a first order approximation of the sensitivity matrix. The CCD measurements (*u, v*) for each LED are related to the tracker coordinate system in the following way:

$$\begin{bmatrix}u \\ v\end{bmatrix} = \begin{bmatrix}-\dfrac{l^{LED}{}_x}{l^{LED}{}_y} \\ -\dfrac{l^{LED}{}_z}{l^{LED}{}_y}\end{bmatrix}$$

where $l^{LED} = (l^{LED}{}_x, l^{LED}{}_y, l^{LED}{}_z)$ is the position of a LED in the tracker coordinate system. The sensitivity matrix for each LED measurement has the form:

$$\frac{\partial\begin{bmatrix}u \\ v\end{bmatrix}}{\partial\begin{bmatrix}l \\ \lambda\end{bmatrix}} = -\begin{bmatrix}1 & u & 0 \\ 0 & v & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & \bigg| & -L^{-1}\left(l^{LED} - l\right)^{x} \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \end{bmatrix}\frac{1}{l^{LED}{}_y}$$

The sensitivities of each LED is combined into a common sensitivity matrix H of size 6x6. The estimate of (Δl,λ) is:

$$\begin{bmatrix}\Delta l \\ \lambda\end{bmatrix} = \left(H^T H\right)^{-1} H^T \left[\Delta u_1 \quad \Delta v_1 \quad \Delta u_2 \quad \Delta v_2 \quad \Delta u_3 \quad \Delta v_3\right]^T$$

where the subscripts in u and v indicate the LED measured. Which can be combined with the coordinate transformation defined above lead to the following Least Squares Estimator of (Δm,μ):

$$\begin{bmatrix}\Delta m \\ \mu\end{bmatrix} = S_2 H^{-1}\left[\Delta u_1 \quad \Delta v_1 \quad \Delta u_2 \quad \Delta v_2 \quad \Delta u_3 \quad \Delta v_3\right]^T$$

Note that, $H^{-1} = \left(H^T H\right)^{-1} H^T$ and H is nonsingular for the nominal SRTM operation configuration. To obtain the mean errors rather than the dynamic errors, we will filter the estimates by a 4th order Butterworth filter with a cut-off frequency of 0.05 rad/s and sampling period of 0.25 s (4 Hz) is used. This filter has the form:

$$y(k) = \sum_{m=k}^{k-4} a_m u(m) - \sum_{n=k-1}^{k-4} b_m y(n)$$

where, y is the filtered version of u, and the a's and b's are the filter coefficients.

The estimator has two stages: initialization, and estimation. The initialization state will choose the gains for the given Outboard Target Assembly target assembly set in use and initialize the estimator state. The estimation state will process the measurements and output filtered estimates. The estimator has to be initialized every time the estimator is turned on or reset, the outboard LED array changes (due to outboard antenna rotation as a part of the deployment), or the tracker measurement dropout. The initialization step consists of reading the gain file if it has changed, selecting the gains, and sorting the tracker measurements. This sorting is necessary because the tracker does not put the measurements in any specific order. The estimation process consists of reading the measurements in telemetry, converting them from pixels to engineering units, applying the matrix gain, and filter the estimates to average the oscillations. Since there are up to four tracker measurements in every telemetry frame, the process will have to be repeated for each measurement.

5.2 Performance Simulations

The onboard algorithm was coded in Matlab and tested using simulated sensor data obtained from the hardware testbed. A test was done to evaluate the nominal mast deployment configuration with attitude hold, and with attitude subject to a series of thruster pulses, one in each axis. The performance of the estimator was tested in a hardware-in-the-loop testbed. The tests evaluated the attitude accuracy of the estimator through comparing the estimator result with independent attitude measurements. In this test, the LED array were driven by a Motion Simulator to simulate a 4 cm circular mast tip motion. Figures 4 shows the X component of position and attitude estimates. The plots show that the estimator tracks the actual dynamics quite accurately. The biases in the estimates are due to misalignments of the tracker and the outboard antenna centroid.

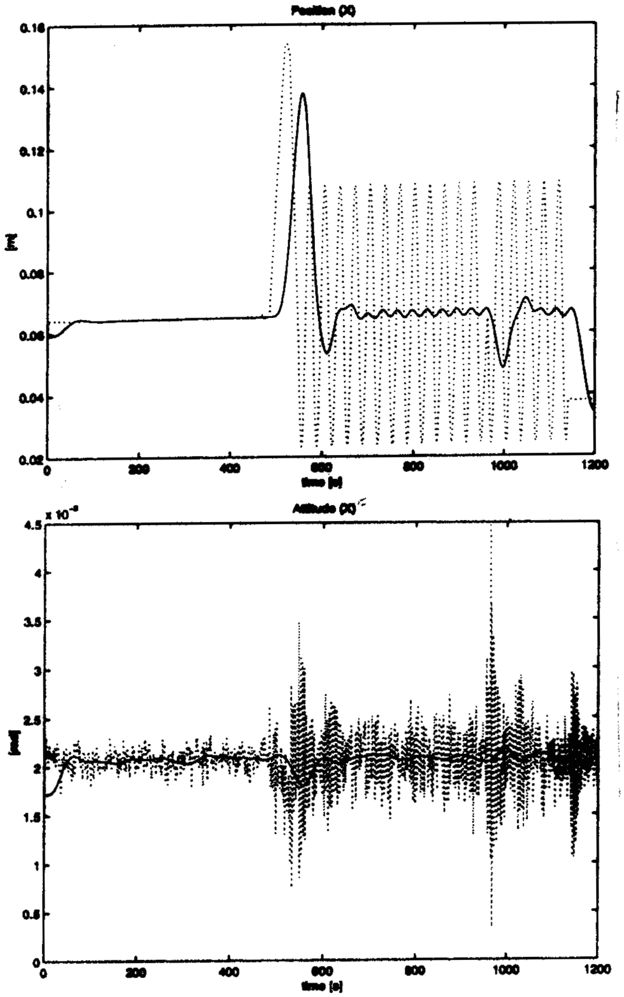


Figure 4 Estimated position and attitude of outboard antenna relative to inboard via onboard software (solid line is the estimate)

6. Estimators for Ground Attitude Reconstruction

6.1 Attitude Determination of Outboard Antenna

The post-flight determination of position and attitude of the Outboard antenna with respect to the Inboard one is carried out by a Kalman filter. To achieve this goal the estimator will use the target tracker, EDMs, and thrusters firing histories. Attitudes between measurement will be propagated based on a reduced dynamic model of the spacecraft.

6.1.1 Estimator Formulation

State Variable Model

To solve the problem of estimating the position and attitude of the outboard antenna with respect to the inboard (m and M as defined above), a discrete time Kalman filter with a predictor/corrector is used to interpolate the measurements stored during the mission. In particular, we implement format. The estimator state is defined as

$$\boldsymbol{x} = \begin{bmatrix} x_p & y_p & z_p & x_a & y_a & z_a \mid m_1 & m_{1r} \mid \cdots & \cdots \mid m_n & m_{nr} \end{bmatrix}^T$$

which is a (6 + 2n x 1) vector, where (x_p, y_p, z_p) is Δm bias (Δm is the error in m), (x_a, y_a, z_a) is μ bias (μ is the error in M), n is the number of flexible modes, m_i is the mode-i magnitude, is the m_{ir} mode-i rate.

The dynamics of the state are modeled as a discrete time state variable model. These equations propagate the state based on thruster firings measurements (u).

$$x(k+1) = \Phi(k+1,k)x(k) + \Psi(k+1,k)u(k) + \Gamma(k+1,k)w(k)$$

$$X(k+1) = \Phi(k+1,k)X(k)\Phi(k+1,k)^T + \Psi(k+1,k)U(k)\Psi(k+1,k)^T + \Gamma(k+1,k)W(k)\Gamma(k+1,k)^T$$

$$z(k+1) = H(k+1)x(k+1) + v(k+1)$$

$$u(k) = P\begin{bmatrix} f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \end{bmatrix}^T(k)$$

such that

$$\Phi(k+1,k) = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & M(k+1,k) \end{array}\right]$$

$$\Psi(k+1,k) = \left[\begin{array}{c} 0 \\ \hline \Psi_M(k+1,k) \end{array}\right] \qquad \Gamma(k+1,k) = I$$

$$M(k+1,k) = e^{A(t_{k+1}-t_k)} \approx I + A(t_{k+1}-t_k)$$

$$U(k) = 0.01u^T u I_{6\times 6}$$

$$A = \left[\begin{array}{cc|cc|cc} 0 & 1 & \cdots & \cdots & 0 & 0 \\ -\varpi_1^2 & a_{11} & \cdots & \cdots & 0 & a_{1n} \\ \hline \vdots & \vdots & \ddots & & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ \hline 0 & 0 & \cdots & \cdots & 0 & 1 \\ 0 & a_{n1} & \cdots & \cdots & -\varpi_n^2 & a_{nn} \end{array}\right]$$

$$W(k) = \left[\begin{array}{c|cc|c|cc} \Sigma_{Thr} & 0 & & \cdots & 0 & \\ \hline \multirow{2}{*}{0} & 0 & 0 & \multirow{2}{*}{$\cdots$} & 0 & 0 \\ & 0 & \rho_{11} & & 0 & 0 \\ \hline \vdots & & \vdots & \ddots & & \vdots \\ \hline \multirow{2}{*}{0} & 0 & 0 & \multirow{2}{*}{$\cdots$} & 0 & 0 \\ & 0 & 0 & & 0 & \rho_{nn} \end{array}\right]$$

$$\Psi_M(k+1,k) = \left(\int_0^{(t_{k+1}-t_k)} e^{A\tau} d\tau\right) \left[\begin{array}{c|c|c} 0 & \cdots & 0 \\ E_1^T b_1 & \cdots & E_1^T b_6 \\ \hline \vdots & \ddots & \vdots \\ \hline 0 & \cdots & 0 \\ E_n^T b_1 & \cdots & E_n^T b_6 \end{array}\right] P \approx (t_{k+1}-t_k) \left[\begin{array}{c|c|c} 0 & \cdots & 0 \\ E_1^T b_1 & \cdots & E_1^T b_6 \\ \hline \vdots & \ddots & \vdots \\ \hline 0 & \cdots & 0 \\ E_n^T b_1 & \cdots & E_n^T b_6 \end{array}\right] P$$

$$H(k+1) = \frac{\partial z}{\partial l\lambda} \frac{\partial l\lambda}{\partial m\mu} \left[I \quad | \; cE_1 \quad 0 \; | \; \ldots \quad \ldots \; | \; cE_n \quad 0 \right]$$

$$z_i(k) = \left[\mu_i^{CCD} \quad v_i^{CCD} \right]^T, i = 1,2,3 \qquad z_4(k) = \Delta v^{EDM}$$

$$\frac{\partial z_i}{\partial l\lambda} = H_{z_i} \text{ and } \frac{\partial l\lambda}{\partial m\mu} = S_2^{-1}$$

where Ξ is a block diagonal matrix that summarizes the force and its torque in the center of mass. Each diagonal entry has a column vector ξ_j. f_i indicates if the thruster i was on or off during that time step, and P is diagonal matrix which contains the magnitudes of forces and torques for each vernier jet.

The state variable model has two parts: one relates to the bias states, and the other to the modal states. The dynamics of these two parts are independent. The bias part does not have any dynamics, therefore the bias states are propagated via an identity matrix. The modal part represents the mast flexible body dynamics with the viscous dampers.

The process noise is zero-mean with intensity *W*. The noise intensity has a diagonal block structure. The first block corresponds to the changes in the biases due to the thermal distortions (orbit to orbit, mission cool down). The rest correspond to the flexible mode perturbations due to model uncertainties. Uncertainties in model parameters and Shuttle perturbations (e.g., crew originated disturbances) appear in the input noise U. For the target tracker, each spot is treated as an independent measurement in order to handle the situation of fewer than 3 spots. Uncertainties in the measurements is represented by the noise V.

EDM latency modification

EDM latency is the time between the determination of the least significant figure of the range measurement and the time tag associate to the measurement. The estimator state is augmented to include the EDM latency τ, which is assumed constant or slowly varying. The state propagation equations will be modified to include the latency, which will be driven by a small white noise to allow for varying latency.

$$\begin{bmatrix} x \\ \tau \end{bmatrix}(k+1) = \begin{bmatrix} \Phi & 0 \\ 0 & 0 \end{bmatrix}(k+1,k)\begin{bmatrix} x \\ \tau \end{bmatrix}(k) + \begin{bmatrix} \Psi & 0 \\ 0 & 0 \end{bmatrix}(k+1,k)u(k) + \begin{bmatrix} \Gamma & 0 \\ 0 & 1 \end{bmatrix}(k+1,k)\begin{bmatrix} w \\ N_\tau \end{bmatrix}(k)$$
$$= \tilde{\Phi}(k+1,k)\tilde{x}(k) + \tilde{\Psi}(k+1,k)u(k) + \tilde{\Gamma}(k+1,k)\tilde{w}(k)$$
$$\tilde{X}(k+1) = \tilde{\Phi}(k+1,k)\tilde{X}(k)\tilde{\Phi}(k+1,k)^T + \tilde{\Psi}(k+1,k)U(k)\tilde{\Psi}(k+1,k)^T + \tilde{\Gamma}(k+1,k)\tilde{W}(k)\tilde{\Gamma}(k+1,k)^T$$

The EDM measurement equation will be corrected to include the latency effect as follows:

$$z_{EDM}{}^{meas} = z_{EDM}{}^{pred} - \tau \frac{\partial z_{EDM}{}^{pred}}{\partial t}$$

where

$$\frac{\partial z_{EDM}{}^{pred}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\partial z_{EDM}{}^{pred}}{\partial l\lambda} \frac{\partial l\lambda}{\partial m\mu} \left[I \quad | \; cE_1 \quad 0 \; | \; \ldots \quad \ldots \; | \; cE_n \quad 0 \right] \hat{x} \right)$$
$$= \frac{\partial z}{\partial l\lambda} \frac{\partial l\lambda}{\partial m\mu} \left[0 \quad | 0 \quad cE_1 \; | \; \ldots \quad \ldots \; | 0 \quad cE_n \right] \hat{x}$$

Therefore, the measurement equations will be modified to as follows:

$$ATT: \quad z_i(k+1) = \left[H \quad 0 \right](k+1)\tilde{x}(k+1) + v(k+1)$$
$$EDM: \quad z_i(k+1) = \left[H \quad -\hat{H}\hat{x} \right](k+1)\tilde{x}(k+1) + v(k+1)$$
$$\hat{H}(k+1) = \frac{\partial z}{\partial l\lambda} \frac{\partial l\lambda}{\partial m\mu} \left[0 \quad | 0 \quad cE_1 \; | \; \ldots \quad \ldots \; | 0 \quad cE_n \right]$$

The augmented state variable model is used in the sequel.

Outboard antenna position and attitude estimates with respect to Inboard antenna

The predictor-corrector Kalman filter is used in the format.

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)r(k+1|k)$$
$$\hat{X}(k+1|k+1) = \left[I - K(k+1)H(k+1) \right]\hat{X}(k+1|k)\left[I - K(k+1)H(k+1) \right]^T + K(k+1)V(k+1)K(k+1)^T$$

where, the estimator gain and the innovation process are defined as:

$$K(k+1) = \hat{X}(k+1|k)H^T(k+1)\left(H(k+1)\hat{X}(k+1|k)H(k+1)^T + V(k+1) \right)^{-1}$$
$$r(k+1|k) = z(k+1) - H(k+1)\hat{x}(k+1|k)$$

for k=0,1,2,…

The time interval for the propagation of the state and measurement equations will vary. The state equation will be propagated from thruster firings times to measurement updates times dictated by the data, and stopping at GPS integer second times to produce outputs. During thruster firings we will propagate at a fixed rate (25Hz). Measurement updates will happen every time we have a new measurement. If we have several measurements at the same time, we will process them sequentially without intervening propagation.

The estimates of the relative position and orientation of Outboard antenna with respect to Inboard antenna can be obtained via the following transformation:

$$\begin{bmatrix} \Delta m \\ \mu \end{bmatrix}(k) = \left[I \quad | \; cE_1 \quad 0 \; | \; cE_2 \quad 0 \; | \; cE_3 \quad 0 \right]\hat{x}(k)$$

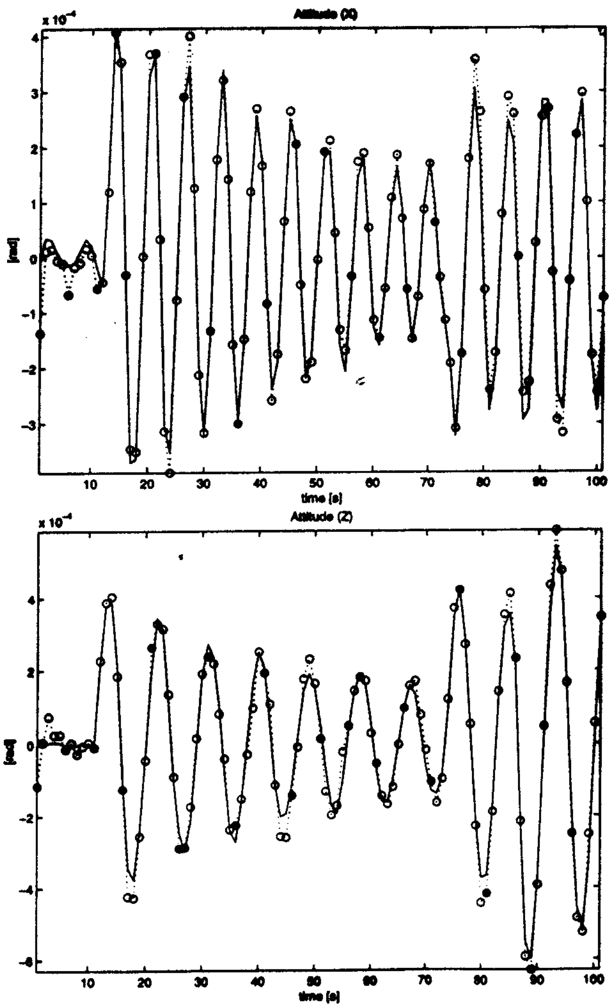
The uncertainty on this estimate is:

$$C_{\begin{bmatrix} \Delta m \\ \mu \end{bmatrix}} = \left[I \quad | \; cE_1 \quad 0 \; | \; cE_2 \quad 0 \; | \; cE_3 \quad 0 \right]\hat{X}(k)\left[I \quad | \; cE_1 \quad 0 \; | \; cE_2 \quad 0 \; | \; cE_3 \quad 0 \right]^T$$

The estimate is output at 1Hz rate, while the state estimates were computed at a faster rate. Therefore we will apply these transformations after resampling the state and covariance estimates.

6.1.2 Performance Simulations

The simulation test corresponds to a 40s of nominal SRTM radar mapping operation (attitude hold). It was simulated with 0.01s step-size and 26 flexible modes. The estimates have been plot at 100 Hz to see in detail the evolution of the estimator. Fig 5 presents plots comparing the simulated dynamics with the estimates of the relative attitude and position of the outboard antenna in X and Z components.



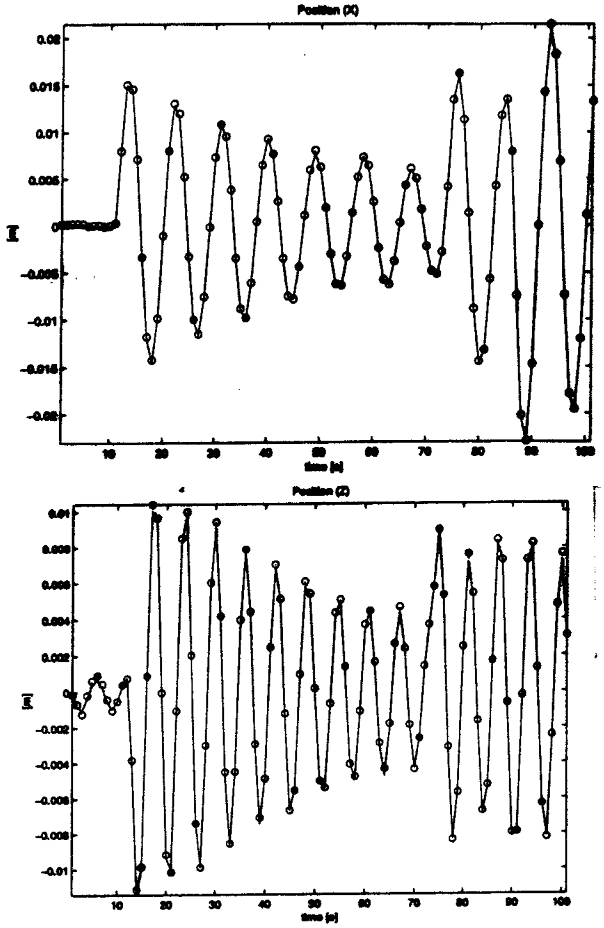


Figure 5 Estimated position and attitude of outboard antenna relative to inboard via ground software

6.2 Attitude Determination of Inboard (main) antenna

The in-board attitude estimator is required to generate and deliver estimates of the orientation of the In-board coordinate System relative to WGS84 and TCN. The attitude is represented by the corresponding quaternion.

6.2.1 Estimator Formulation

Attitude measurements are provided by the star tracker in the form of a quaternion q describing the orientation of the star tracker frame relative to an inertial coordinates (J2000). Three-axis variances are also provided representing the residual error of the measurement with respect to a predicted attitude. These variances will be used to construct a diagonal measurement noise covariance matrix for the attitude estimator. Before further processing the attitude quaternion, a correction for velocity aberrations is applied.

Rate Measurements

The plant and measurement noise processes are white with $E\{w_k\}=0$, $E\{w_k w_j^T\}=Q_k$, $E\{v_k\}=0$, and $E\{v_k v_j^T\}=R_k$.

Propagation Between Updates (Prediction)

The filtered rate is evaluated by: $\hat{\omega}_k = \omega_k^m - \hat{b}_k$, and state propagation is done by: $\hat{x}_{k|k-1} = \Phi_{k-1}\hat{x}_{k-1|k-1}$; $\hat{x}_{k-1|k-1} \equiv 0$. This relation is presented for completeness only since it, obviously, implies that $x_{k|k-1} \equiv 0$.

Covariance Propagation:

$$P_{k|k-1} = \Phi_{k-1}P_{k-1|k-1}\Phi_{k-1}^T + Q_{k-1}$$

Quaternion Propagation under small angle approximation is performed by:

$$\hat{q}_{k|k-1} = \begin{bmatrix} \frac{T}{2}\hat{\omega}_k \\ 1 \end{bmatrix} \hat{q}_{k-1|k-1}$$

Measurement Update (Correction) equations are given by:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(\delta\theta_k^m - H\hat{x}_{k|k-1})$$

$$P_{k|k} = (I - K_k H)P_{k|k-1}$$

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R_k)^{-1}$$

$$\hat{q}_{k|k} = \delta\hat{q}_k \hat{q}_{k|k-1}; \quad \delta\hat{q}_k = \begin{bmatrix} \frac{1}{2}\delta\hat{\theta}_{k|k} \\ 1 \end{bmatrix}$$

$$\hat{b}_{k|k} = \hat{b}_{k|k-1} + \delta\hat{b}_{k|k}$$

6.2.3 Performance Simulations

The estimator has been tested with simulated data. Fig 6 shows the estimated inboard antenna attitude quaternion for one orbit.

7. Identification of Mast Frequency and Damping

The system identification task is motivated by two reasons: (i) the need for fine tuning certain notch filters in the attitude controller of the space shuttle, and (ii) verifying deployment and appropriate operation of the SRTM mast dampers. For the former, it is required to estimate the modal frequencies for the mast, while

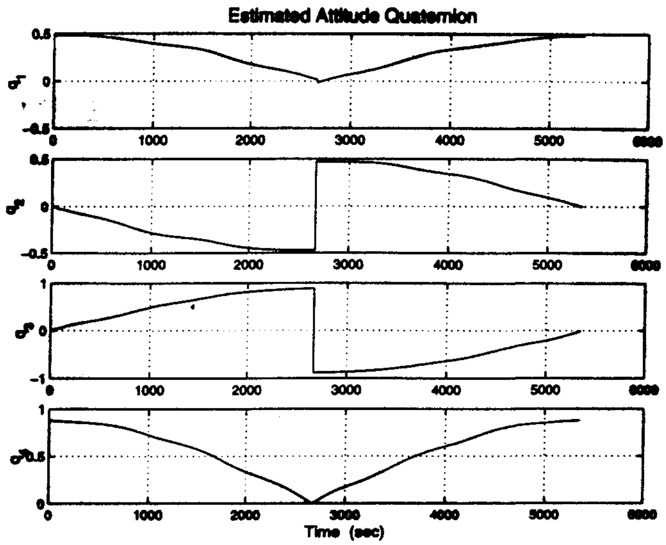


Figure 6 Estimated inboard antenna attitude in inertial for one orbit

the latter requires approximate determination of the damping coefficients.

From a dynamics point of view, the mast is treated as a linear time-invariant system that could be represented by a state-space realization (*A,B,C,D*). The objective of the present task is to generate a minimal realization $(\hat{A},\hat{B},\hat{C},\hat{D})$ which approximates the modal characteristics of the original system.

Two experiments will be conducted immediately after deployment of the mast. The first experiment will be performed with the dampers caged (low-damping) and the second with the dampers uncaged (high-damping). Each experiment will result in a data set of, approximately, fifteen minutes duration.

Freedom in experiment design is rather limited due to constraints in on-board sensing and actuation hardware. Specifically, existing system identification algorithms have been developed under the assumption of persistent excitation, sufficiently large number of measured parameters and observation data sets of arbitrary length. In the SRTM case, measurements are limited to displacements and rotations of the mast out-board tip relative to the in-board coordinate system, generated by processing target tracker raw data. Excitation of the structure will be effected by pulsing the thrusters, therefore, an appropriate firing sequence must be

The IRU may consist of m gyro sense axes defined, in body coordinates by unit vectors $g_i \in R^3$, $i = 1,2,\ldots,m$. Three of the axes $\{m_1, m_2, m_3\}$ are chosen as primary, each sensing the corresponding projection of the spacecraft inertial rate $\omega \in R^3$, $\omega_g = G_p \omega$, where $\omega_g = [\omega_{g_1} \quad \omega_{g_2} \quad \omega_{g_3}]^T$ and

$$G_p = \begin{bmatrix} g_{m_1}^T \\ g_{m_2}^T \\ g_{m_3}^T \end{bmatrix}$$

The matrix $G_p \in R^{3\times3}$ consists of linearly independent rows, therefore, it is non-singular. Furthermore, G_p is, in general, non-orthogonal. Ideally then we should get $\omega = G_p^{-1}\omega_g$. Next let G denote the *true* orientation of the three axes so that $G = G_p(I + \Delta G)$ where ΔG is a matrix of small angles representing the gyro axis alignment errors. Furthermore let $K_g = diag\{k_1, k_2, k_3\}$ be a diagonal matrix of scale factor errors. Then the gyro axis measurements may be defined as

$$\begin{aligned} \omega_g^m &= (I + K_g)G\omega + b_g + n_g \\ &= G(I + G^{-1}K_gG)\omega + b_g + n_g \end{aligned}$$

where b_g and n_g are the rate bias and noise terms, respectively. A measurement in body axes may now be defined as

$$\begin{aligned} \omega_m &= G_p^{-1}\omega_g^m \\ &= \omega + b + n \end{aligned}$$

where
$$\begin{aligned} b &= G_p^{-1}b_g + (G^{-1}K_gG + \Delta G)\omega \\ n &= G_p^{-1}n_g \end{aligned}$$

<u>*Linearized State Equations*</u>

Let q and $\hat{q}$ be the true and estimated attitude quaternions, respectively, related by

$$q = \delta q \hat{q}; \qquad \delta q = \begin{bmatrix} \frac{1}{2}\delta\theta \\ 1 \end{bmatrix}$$

where δq represents the correction defined as $\delta q = q\hat{q}^*$ with (*) used to represent the conjugate quaternion. Differentiating with respect to time and maintaining first-order terms, it can be shown that

$$\begin{aligned} \delta\dot{\theta} &= -\hat{\omega}^\times \delta\theta + \omega - \hat{\omega} \\ &= -\hat{\omega}^\times \delta\theta - \delta b + n_1 \end{aligned}$$

or, in matrix form,

$$\begin{bmatrix} \delta\dot{\theta} \\ \delta\dot{b} \end{bmatrix} = \begin{bmatrix} -\hat{\omega}^\times & -I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta\theta \\ \delta b \end{bmatrix} + \begin{bmatrix} n_1 \\ w_2 \end{bmatrix}$$

or, $\dot{x}(t) = A(t)x(t) + w(t)$. Assuming that the rate estimate remains constant within one sampling interval $(t_k \le t \le t_k + T)$ the state equation may be discretized to obtain

$$\begin{aligned} x_k &= \Phi_{k-1}x_{k-1} + w_{k-1} \\ \Phi_k &= I + A(\hat{\omega}_k)T \\ w_k &= \int_0^T e^{A(\hat{\omega}_k)(T-\tau)} w(\tau)d\tau \end{aligned}$$

<u>*The Observation*</u>

The true, measured, and estimated quaternions are related by

$$q^m = \delta q^m \hat{q}; \qquad \delta q^m = \begin{bmatrix} \frac{1}{2}\delta\theta^m \\ 1 \end{bmatrix}$$

$$q^m = \delta q_v q; \qquad \delta q_v = \begin{bmatrix} \frac{1}{2}\delta v \\ 1 \end{bmatrix}$$

$$q = \delta q \hat{q}; \qquad \delta q = \begin{bmatrix} \frac{1}{2}\delta\theta \\ 1 \end{bmatrix}$$

where δq^m represents the measurement residual, δq_v represents the measurement error due to noise, and, δq the estimation error. It can be easily shown that

$$\begin{aligned} \delta\theta^m &= (I - \frac{1}{2}v^\times)\delta\theta + v \\ &\approx \delta\theta + v \end{aligned}$$

or, in terms of the previously defined discrete-time state vector,

$$\delta\theta_k^m = Hx_k + v_k$$

where $H = [I_{3\times3} \quad 0_{3\times3}]$, and the independent measurement is generated by

$$\delta\theta_k^m = 2S(q_m^k \hat{q}_{k|k-1}^*) \text{ with } S = [I_{3\times3} \quad 0_{3\times1}].$$

<u>6.2.2 The Estimator</u>

Estimation of the state is performed by taking a standard extender Kalman filtering approach. The system model, obtained by linearizing the state equations with respect to the current attitude and rate estimates $\{\hat{q}_k, \hat{\omega}_k\}$, is described by

$$\begin{aligned} x_k &= \Phi_{k-1}x_{k-1} + w_{k-1} \\ \delta\theta_k^m &= Hx_k + v_k \end{aligned}$$

designed, which will ensure the richest (in frequency content) possible excitation of the structure.

7.1 The Identification Approach

At the time instant t, let $u(t) \in R^m$ denote the excitation input (either force or commanded acceleration) applied to the mast and $y(t) \in R^r$ a vector of measured (output) states. The observation is constructed such that it contains sufficient information to characterize both the bending and torsional motion of the mast. The two signals are recorded at sampling times $\{t_k\}$ $k = 1,2,\ldots$, thus generating the sequences $\{u_k\} \in R^m$ and $\{y_k\} \in R^r$. If thruster firing information is not available within the desired time resolution, it may be replaced with either of the commanded force or acceleration profiles. The output sequence samples are, as stated earlier, generated by processing the raw target tracker measurements.

7.2 The System Identification Package

A system identification software package has been assembled. It utilizes existing state-of-the-art algorithms and is implemented in MATLAB. Specifically, the package consists of three subroutines:

1. A data processing algorithm to convert target tracker data to three-axis (small) angles,
2. the Observer/Kalman Filter Identification (OKID) algorithm, and
3. the Eigensystem Realization Algorithm (ERA)

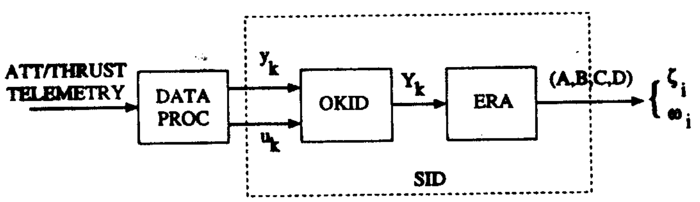


Figure 7 Identification process of modal frequency and damping

The identification process is shown in the functional block diagram of Fig 7. The input and measured output sequences $\{u_k\}$ and $\{y_k\}$ are pre-processed and converted to the appropriate physical variables and engineering units. The processed data are then introduced to the OKID algorithm which estimates a set of Markov parameters. A clarification on the nature of these parameters is in order.

The input-output relation for any linear time-invariant dynamic system with state-space realization (A,B,C,D) is completely characterized by

$$y_k = \sum_{i=0}^{k} Y_k u_{k-i}$$

where $Y_0 = 0$ and $Y_k = CA^{k-1}B$, $k = 1,2,\ldots$ are the *System Markov Parameters*. The OKID algorithm does not return system Markov parameters, instead, it employs a Kalman filter approach to generate the set $\{Y_k\}$ of *Kalman Filter Markov Parameters*. It also returns the corresponding Kalman filter gain K. Further processing is, therefore, required in order to obtain the *system* characteristics. This is accomplished by utilizing the ERA to generate a minimal realization $(\hat{A},\hat{B},\hat{C},\hat{D})$ based on the sequence $\{Y_k\}$ estimated by OKID. Notice that matrices C and D are the same as the corresponding 'true system' matrices.

The remaining two matrices $\hat{A}$ and $\hat{B}$ are obtained as

$$\hat{A} = \bar{A} + KC$$

$$\hat{B} = \bar{B} + KD$$

The desired frequency and damping information is contained in $\hat{A}$

.

True System Damping (%)	Identified System Damping (%)
0.500	5.0365e-01
0.500	5.1036e-01
0.500	4.9071e-01
0.500	4.6046e-01
0.500	4.8144e-01
Freq (Hz)	Freq (Hz)
0.093	9.2995e-02
0.121	1.2101e-01
0.140	1.4001e-01
0.481	4.8072e-01
0.731	7.3095e-01

To demonstrate this identification method, a five-mode system was simulated with modal frequencies, damping and plant and measurement noise at the values expected in SRTM. The following table makes a comparison of the true modal characteristics, and those generated by the identification algorithm. Furthermore, Fig 8 shows the true and identified system responses which appear to be in agreement. The input excitation is normalized to unity and was applied at time 4 sec over 1 second duration.

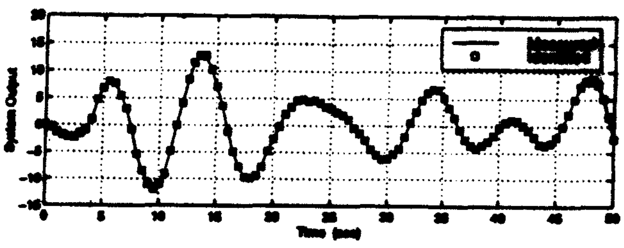


Figure 8 SRTM system identification: true and identified system response

8. Conclusion

SRTM is the first mission to provide high accuracy near-global topographic coverage of the earth's land surface using a long-baseline (60 m) interferometry approach. The attitude determination system is designed to demonstrate the application of attitude determination techniques to interferometric baseline reconstruction. An onboard estimator has been designed to determine the attitude of the outboard radar antenna relative to the inboard (main) antenna for mast deployment verification and alignment. Both onboard and ground estimators and their performance have been designed and simulated, and have been shown to satisfy all the attitude determination requirements needed for this mission.

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Table 1

Inboard antenna attitude errors in Inertial coordinates.

Stability/noise	**Budgeted error 1.6 sigma in arcsec** X (roll)	Y (pitch)	Z (yaw)
Star tracker/IRU attitude determination accuracy	1.29	3.88	1.03
Star tracker to Inboard stability (structural, thermal etc.)	1.0	7.0	7.0
Inertial (TCN) coordinates with respect to Earth (ECI) coord.	0.03	0.03	0.03
Timing jitter	0.01	0.01	0.01
11 day relative accuracy (RSS)	1.7	8.1	7.1
Requirement	5.0	15.0	15.0

Table 2

Outboard antenna attitude errors relative to Inboard.

Stability/noise	**Budgeted error 1.6 sigma in arcsec** X (roll)	Y (pitch)	Z (yaw)
Outboard targets to outboard antenna stability (structural, thermal etc.)	1.0	1.0	1.0
Target tracker to Inboard antenna stability (structural, thermal etc.)	1.0	1.0	1.0
Target tracker to Outboard targets accuracy (1-sec accuracy)	38.5	58.1	37.9
Timing jitter	2.16	1.62	0.48
11 day relative accuracy (RSS)	39.8	59.7	38.5
Requirement	100.	120.0	120.0

Table 3

Outboard origin position error relative to Inboard

Stability/noise	**Budgeted error 1.6 sigma in mm** X (roll)	Y (pitch)	Z (yaw)
Outboard targets to outboard antenna location stability (structural, thermal etc.)	0.25	0.25	0.25
target tracker /EDM to inboard antenna location stability (structural, thermal etc.)	0.5	0.5	0.5
Target tracker/EDM to Outboard targets accuracy	0.51	1.37	0.58
Timing jitter	0.18	0.0	0.18
11 day relative accuracy (RSS)	0.94	1.49	0.98
Requirement	8.7	2.0	1.5

Table 4 Coordinate Transformations.

${}^{ICS}T^{ATT} = \left\{A^{-1};a\right\}$	A a	Rotation from Inboard (ICS) to Target Tracker (ATT) coord. Vector from Inboard to Target Tracker in Inboard coordinates.
${}^{OCS}T^{OTA} = \left\{D^{-1};d\right\}$	D d	Rotation from Outboard-mast tip (OCS) to Outboard Target Assembly coordinates (OTA) Vector from Outboard-mast tip to Outboard Target Assembly in Outboard coord.
${}^{ATT}T^{OTA} = \left\{L^{-1};l\right\}$	L l	Rotation from Target Tracker to Outboard Target Assembly coord Vector from Target Tracker to Outboard Target Assembly in Target Tracker coordinates.
${}^{OCS}T^{ICS} = \left\{M^{-1};m\right\}$	M m	Rotation from Inboard to Outboard-mast tip coordinates Interferometric baseline i.e. vector from Inboard to Outboard-mast tip in Inboard coordinates.